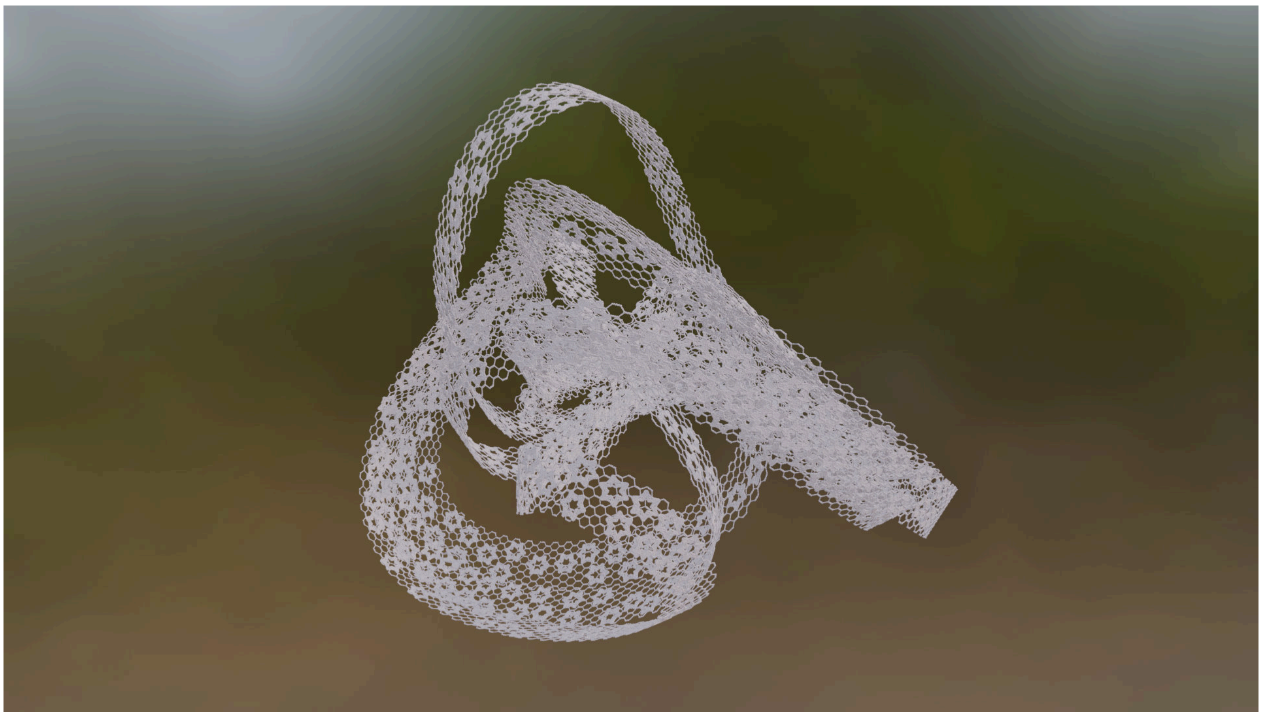
JFIG Concours: Geometry Nodes

Halvorsen Lace



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1 Introduction

Calais has a long history of lace making. The city became a major center in the nineteenth century when machine lace grew in quality and scale. Calais lace is known for a fine tulle base and rich patterns. The structure is light, regular, and strong enough to hold complex shapes.

In this work, lace is modeled in a parametric way. First, we build the tulle from one curve that can extend to any length. Second, we generate a rosette patch by a direct set of equations in (u, v). Third, we place many copies of this patch on the tulle using Poisson disk sampling. Last, we bend the full lace along a Halvorsen attractor to simulate drape and fold. All steps expose clear parameters, so style and density can be changed at any time.

2 Method Overview

The workflow has four stages:

1. **Tulle thread (infinite):** a repeating unit of four segments (vertical line, right-leaning line at 60°, vertical helix, left-leaning line at 60°). A modulo selector picks the active segment on each unit interval. The thread can extend to any length by growing the parameter domain.

2. **Tulle sheet:** many horizontal copies of the thread. Every other copy rotates by 180°. This gives a regular, hexagon-like mesh similar to real tulle.

3. **Rosette patch:** a simple parametric system in (u, v) yields a rosette in the plane. The patch has a radius function with K-fold symmetry and a width offset ϕ along the radial direction to make a ring band around the main curve.

4. **Placement and bending:** we scatter the rosette patches across the tulle with Poisson disk instancing, then bend the whole lace along a Halvorsen attractor curve.

3 Tulle Construction

3.1 Repeating Unit and Modulo Selection

Let $t \in \mathbb{R}$ be the running parameter along the thread. Define $s = t \bmod 1 \in [0, 1)$ as the phase within each unit, and $n = \lfloor t \rfloor$ as the unit index. Each unit has four ordered segments, each of unit length: 1. a vertical straight line; 2. a straight line leaning to the right by 60°; 3. a vertical helical segment; 4. a straight line leaning to the left by 60°.

We split $[0, 1)$ into four equal sub-intervals of length $\frac{1}{4}$ and select the active formula by the value of s. Let $L = 1$ be the unit length. Let the base point of unit n be $\mathbf{B}_n \in \mathbb{R}^3$, defined by summing the end of the previous unit. Then the piecewise curve is

$$\mathbf{r}(t) = \begin{cases} \mathbf{B}_n + \mathbf{r}_{\text{vert}}(\tau), & s \in [0, \frac{1}{4}), \\ \mathbf{B}_n + \mathbf{r}_{+60}(\tau), & s \in [\frac{1}{4}, \frac{1}{2}), \\ \mathbf{B}_n + \mathbf{r}_{\text{helix}}(\tau), & s \in [\frac{1}{2}, \frac{3}{4}), \\ \mathbf{B}_n + \mathbf{r}_{-60}(\tau), & s \in [\frac{3}{4}, 1), \end{cases} \qquad \tau = 4s \in [0, 1).$$

The four segment maps are:

$$\mathbf{r}_{\text{vert}}(\tau) = (0,\, 0,\, L\tau), \qquad \mathbf{r}_{+60}(\tau) = \left(\frac{L}{2}\tau,\, 0,\, \frac{\sqrt{3}L}{2}\tau\right),$$
$$\mathbf{r}_{\text{helix}}(\tau) = (R\cos(2\pi\tau),\, R\sin(2\pi\tau),\, H\tau), \qquad \mathbf{r}_{-60}(\tau) = \left(-\frac{L}{2}\tau,\, 0,\, \frac{\sqrt{3}L}{2}\tau\right).$$

Here R is the helix radius and H is the helix pitch over one unit. The base transform $\mathbf{B}_n$ advances by the end of the previous segment set, so the thread continues without gaps. Because the selector uses $s = t \bmod 1$, the thread can grow to any length by extending t.

3.2 Forming the Tulle Sheet

To make the sheet, we copy the thread along the x-axis with spacing Δx. For the jth copy, we apply a rotation of 180° around the z-axis if j is odd, and no rotation if j is even:

$$\mathbf{R}_j = \begin{cases} \mathbf{I}, & j \text{ even}, \\ \text{Rot}_z(\pi), & j \text{ odd}. \end{cases}$$

This alternating rotation makes a regular, hexagon-like linkage between neighbors and produces a valid tulle layout. In Geometry Nodes, this is an *Instance on Points* step with a simple index-driven switch for the 180° rotation. The image of tulle and a close up of fine structure in shown in Figure 1

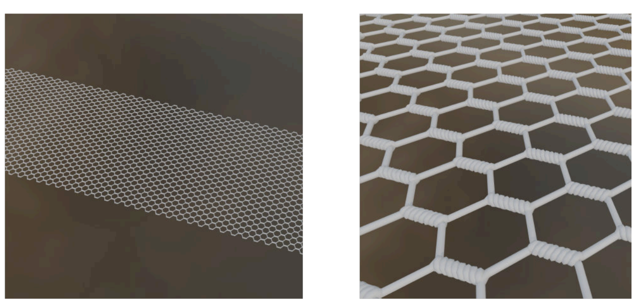

Figure 1: Tulle

4 Rosette Patch Definition

We define the rosette patch by the following parametric system. Let Index be the point index on a (U Count) $\times$ (V Count) grid:

$$
\begin{aligned}
u &= \frac{\text{Index} \bmod U\ \text{Count}}{U\ \text{Count} - 1}, \\
v &= \frac{\left\lfloor \frac{\text{Index}}{U\ \text{Count}} \right\rfloor}{V\ \text{Count} - 1}, \\
\theta &= 2\pi u, \\
r &= R_0 + A\cos(K\theta), \\
\phi &= (v - 0.5)\,\text{Width}, \\
x &= (r + \phi)\cos\theta, \\
y &= (r + \phi)\sin\theta, \\
z &= 0.
\end{aligned}
$$

Parameters:

- R_0: base radius,
- A: radial amplitude,
- K: number of lobes,
- Width: thickness of the band around the rosette.

This patch is a band around a K-fold rose curve. The (u, v) grid gives clean control of sampling along the ring (via u) and across the ring width (via v). In Geometry Nodes, these formulas map to *Index*, *Modulo*, *Floor*, *Multiply-Add*, and standard trigonometric nodes.

5 Placing the Pattern on the Tulle

We place many rosette patches on the tulle with Poisson disk instancing:

This yields even spacing without overlap. The density is set by d_{min} and by the base size (R_0, A, Width). Because the rosette patch is parametric, the same node group can scale and rotate instances without loss of shape quality. Figure 2 shows some lace with different patterns.

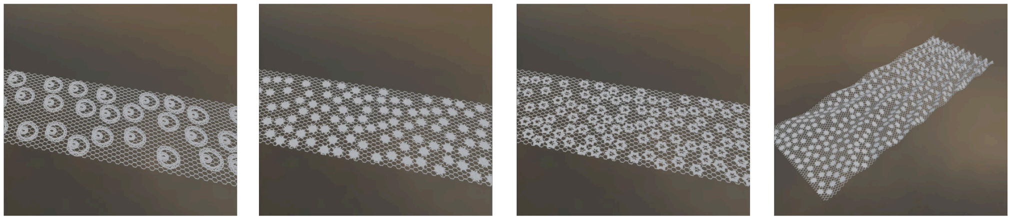

Figure 2: Tulle with Patterns

6 Bending Along a Halvorsen Attractor

6.1 Guide Curve

We use a Halvorsen attractor to get a smooth guide curve with rich bends. The system is

$$\begin{aligned}\dot{x} &= -ax - 4y - 4z - y^2,\\ \dot{y} &= -ay - 4z - 4x - z^2,\\ \dot{z} &= -az - 4x - 4y - x^2.\end{aligned}$$

We integrate this system with a fixed time step to get a point set $\{(x_i, y_i, z_i)\}$. We then rescale to the target size and convert to a Blender curve.

6.2 Deformation Setup

There are two simple options in Geometry Nodes:

1. **Curve Deform path:** convert the attractor points to a *Curve*, align the lace to a local axis, and apply a curve deform modifier or an equivalent node setup.
2. **Mapping by arclength:** compute arclength s along the attractor and map each vertex of the lace (with a precomputed longitudinal parameter) to a position on the curve with *Sample Curve*. A *Tangent* from the curve gives a Frenet-like frame for orientation.

Key parameters are the attractor coefficient a, the deformation strength, and the total arclength window used. Results are shown in Figure 3.

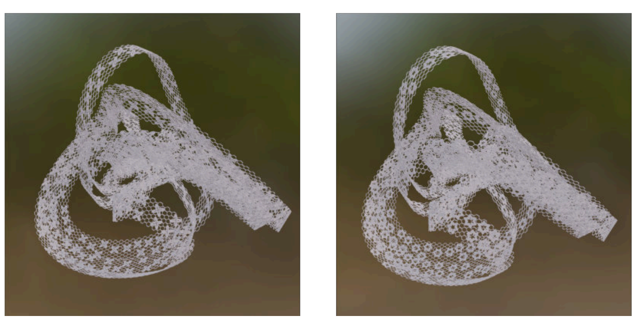

Figure 3: Halvorsen Lace